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On the Network Effect in Web 2.0 Applications

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Abstract

In recent years, the proliferation of the world wide web has lead to an increase in a number of applications such as search, social networks and auctions, whose success depends critically upon the number of users of that service. Two examples of such applications are internet auctions and social networks. One of the characteristics of online auctions is that a successful implementation requires a high volume of buyers and sellers at its website. Consequently, auction sites which have a high volume of traffic have an advantage over those in which the volume is limited. This results in even greater polarization of buyers and sellers towards a particular site. The same is true for social networks in which greater use of a given social network increases the use from other participants on the network. This is often referred to as the "network effect" in a variety of interaction-centric applications in networks. While this effect has qualitatively been known to increase the value of the overall network, its effect has never been modeled or studied rigorously. In this paper, we construct a Markov Model to analyze the network effect in the case of two important classes of web applications. These correspond to auctions and social networks. We show that the network effect is very powerful and can

*This paper is an extended version of (Aggarwal and Yu, 2009). This is a pre-print version of a journal article, which will appear in *Electronic Commerce Research and Applications*.

result in a situation in which an auction or a social networking site can quickly overwhelm its competing sites. Thus, the results of this paper show the tremendous power of the network effect for Web 2.0 applications.

1 Introduction

With the increasing use of the world wide web, a number of applications have been developed which are critically dependent upon their adoption by a large number of users. Examples of such applications include *online auctions, social networking sites, and product recommendation sites*. A key characteristic of these sites is that the presence of a larger number of users increases the value of the site for all other users. A number of popular sites such as $Ebay$[1], and $Priceline.com$[2] routinely conduct auctions on the web in order to match buyers and sellers over a variety of products. Similarly, a number of social networking sites such as *Facebook*[3], *Twitter*[4] and *LinkedIn*[5] have developed over time. Consequently, a number of papers have been written in recent years which study the dynamics of applications such as auctions and social networking (Alsemgeest et al. , 1998; Bapna et al. , 2000; Klein and O'Keefe, 1999; Lucking-Reiley, 2000; Shah et al. , 2003; Van Heckjk and Vervest, 1998).

In many web applications, the success of the site highly depends upon the number of users at the site. For example, a social networking site is not very useful if it has a small number of users. Similarly, an auction site is useful only if it has a large number of buyers and sellers who are ready to perform transactions with one another. Therefore the value of these sites greatly increase with the number of users. This is known as the *network effect* (Shapiro and Vairian, 1998). The network effect of the auction system continues to grow in a self-sustaining way with site popularity. Such a network effect is also present in any system such as *community-based* search engines[6] in which the quality of the results can depend upon the number of users utilizing the system. For the case of social networks, dense networks have a clear advantage, since it leads to rapid dissemination of information across the network (Chakrabarti et al. , 2008; Wasserman and Faust, 1994). Large

[1] http://www.ebay.com
[2] http://www.priceline.com
[3] http://www.facebook.com
[4] http://www.twitter.com
[5] http://www.linkedin.com
[6] http://www.alexa.com

social networks have seen a rapid increase in densification and number of participants in recent years (Leskovec et al. , 2005). We note that the exact value, sustainability and impact of the network effect deeply depends upon how it is leveraged for a particular application. In general, the network effect is likely to be experienced in any application where the value of of the network depends upon the level of interaction between the different users.

We note that our analysis pertains to the case of web applications in which the site acts as an *intermediary* to facilitate the interaction between different participants, rather than being a participant itself. For example, in the case of online auctions, the product is sold by an independent seller who is charged a fee for the transaction. This is certainly the case for current auction models such as $Ebay$. While some online auctioneers (such as Policeauctions.com) directly auction their own items[7], this is not the model that we wish to analyze in this paper. This is because the latter model does not play a *matching role* between many buyers and sellers, which is critical for the network effect. Correspondingly, such sites also have less capacity in attracting diverse buyers to their site, or are limited to buyers interested in a particular segment of the fragmented marketplace. *In this paper, we are directly looking only at web applications in which the aim of the application is to act purely as an intermediary to facilitate the interaction between different users.* Our analysis provides an understanding of the competitive dynamics of different applications attracting users to their site, and its corresponding self-sustaining effect.

We note that network interactions can typically be of two types.

- In the first case, the interactions may occur only between certain categories of participants such as buyers and sellers. Examples of such applications include online auctions.
- In the second case, the interaction may occur between any pair of participants at the site. This is in fact the more general case, and encompasses may different kinds of applications such as social networks, instant messaging systems, and chat forums.

[7]This site sells unclaimed items from police recovery efforts of stolen or lost items.

This paper will analyze the network effect for both kinds of applications. First, we will analyze the first case with the use of web auctions as the relevant scenario. Then, we will study the more general case of interactions between any pair of users. For this case, we will study the example of social networks. Thus, the two different cases provide an understanding of different kinds of networks.

The analysis of the network effect helps us understand the dynamics of the phenomenon, as well as the implications for social networking sites in terms of how it should be managed. Specifically, our analysis of the network effect provides an understanding of the following aspects:

- Our analysis provides an understanding of the different factors which are most likely to affect the dominance of one site over another and vice-versa.
- The understanding of the factors, which is provided by our analysis, also helps in designing effective methods for site administrators to tailor their applications, so as to either use or avoid the network effect. In particular, it helps provide an understanding to the secondary players how to grow their network in the presence of a dominant player. In a later section, we will discuss the implications of the results of this paper on network management.

This paper is organized as follows. In the next section, we will discuss the network effect for online auctions. We will construct a Markov Model which determines the equilibrium stability for online auctions. We will also examine some characteristics of this model, the corresponding interpretation, and the equilibrium stability. In section 3, we will study the second case, with the use of social networks as the relevant application. Section 4 is the discussion of implications of the results of this paper on network management. Section 5 will illustrate some analytical simulations which illustrate the power of the network effect. Section 6 will present the conclusions and summary.

1.1 Theoretical Background

The earliest studies on the network effect were found in (Shapiro and Vairian, 1998) which contains a mostly qualitative discussion of the effects of network size on telecommunication networks. In particular, a law known as *Metcalfe's law* was first proposed by Robert Metcalfe in regard to the ethernet, and later formulated by George Gilder in 1993 (Shapiro and Vairian, 1998) with regard to larger telecommunication networks. This law states that the value V of a telecommunication network is proportional to the square of the number of nodes n in the network. In other words, we have:

$$V = C \cdot n^2 \tag{1}$$

Here C is a constant of proportionality. We note that the Metcalfe's law was proposed under the qualitative assumption that the value of the network was dependent upon the number of *potential* interactions between the different network entities. Since the number of potential interactions is proportional to the square of the number of nodes, Metcalfe's law also proposes this as a thumb rule for measuring the value of the network. A different approach (Beckstrom, 2009) proposes the value of the network in terms of its effect on an individual user. This law known as Beckstrom's law (Beckstrom, 2009; Buley, 2009) defines the value of a network in terms of the sum of the individual values of the transactions across all the different users. For example, consider a user who buys their items annually from different networks for 1000 dollars. After the addition of Amazon to the network, the price of the same set of items reduces to 600 dollars. Then, the value of the Amazon to the user is 400 dollars. By summing up this saving across different users, we can determine the value of Amazon to the network. Thus, if $R_i(j)$ and $R'_i(j)$ be the value of transactions of the ith user in the presence and absence of the jth user respectively, then the value V^j of j to the network is given by:

$$V^j = \sum_i (R_i(j) - R'_i(j)) \tag{2}$$

The total value V of the network itself is a summation of the value of each of the network entities. Therefore, we have:

$$V = \sum_j V^j \quad (3)$$

For cases, in which we wish to compute these values for a long time horizon in the future, it is possible to compute the present-value of the network by summing up these values over different time periods, with an appropriate time-discount factor. We note that both Metcalfe's law and Beckstrom's law provide thumb rules to measure the value of the network, but they do not provide an understanding of the network effect dynamic between *competing* networks.

One interesting aspect of the network effect is that the local structure of the network plays a key role in terms of who benefits from whom (Sundarajan, 2007). This is because in the case of many products, a user may not be interested just by the size of the user base in general, but also directly by the decision of the subset of users which are connected to them. This is because these connected individuals form the peer group which has the greatest influence on that particular user. A mathematical model and theoretical analysis of the concept of local network effect is provided in (Sundarajan, 2007). However, this particular analysis is not so much dependent upon the global size of the network itself, but rather on the *specificity* of who may influence whom. It also does not analyze the dynamics of competing networks in terms of consumer behavior.

The network effect is not just relevant for consumer applications, but also a variety of social networking applications in which the social effects can be used in order to improve the underlying service. For example, a variety of search engines such as Google and Alexa utilze user-behavior in order to provide more effective search results. Methods to maximize and evaluate the economic value of networks are discussed in (Buley, 2009; Economides, 1996). A discussion of the network effect in the context of a wide variety of different network and ecommerce-based applications is provided in (Alsemgeest et al. , 1998; Bapna et al. , 2000; Klein and O'Keefe, 1999; Lucking-Reiley, 2000; Shah et al. , 2003; Van Heckjk and Vervest, 1998). None of these papers however provide an understanding of the dynamics of user behavior in the context of multiple networks. This paper provides the first such theoretical study.

2 The Network Effect in Web Auctions

In this section, we will model the network effect in online auctions. In order to do so, we will construct a Markov Model which relates the behavior of buyers and sellers in an online environment. The reason for using a Markov chain is that it tracks the dynamics of the network in the form of a number of probabilistic states which transition between each other. Since the membership of the user in a network can be treated as a state, this can be effectively simulated by a dynamic Markov Model. Clearly, the process, in which users dynamically join and leave networks is a probabilistic process which can be dynamically simulated. Markov chains provide an effective tool for this simulation, especially in terms of the steady-state behavior of the network.

The basic assumptions in the model are as follows:

- The auctioneer acts only as an intermediary agent during the selling process, and is not a direct party to the transaction. Therefore, buyers and sellers are only interested in the most efficient and cost effective transaction.
- A greater number of buyers provides a more effective and cost efficient transaction to the seller. Therefore, a seller is more likely to choose a particular auctioning agent, if it provides access to a greater number of buyers.
- A greater number of sellers provides a more effective and cost efficient transaction to the buyer. Therefore, an auction with a larger number of sellers is also more attractive to the buyer.

We note that the above observations serve as the source of the network effect in online auctions. It is clear that auctions with a larger number of buyers are likely to attract more sellers and vice-versa. This may result in *defection behavior* from one auction to another. This defection behavior creates a self-sustaining effect which results in one dominant auction crowding out the others rapidly. Even in situations in which multiple auctions exist with an equal amount of market dominance, we

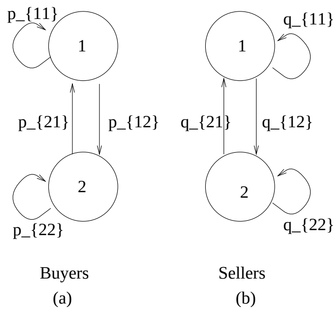

Figure 1: Static Markov Models for Buyer and Seller Defection

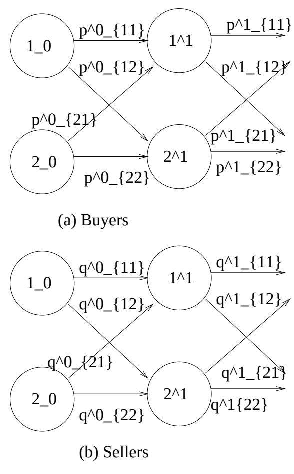

Figure 2: Dynamic Markov Models for Buyer and Seller Defection

will see that the nature of the equilibrium is stochastically unstable, and will lead to dominance of one auction over time.

In order to formally model the propensity of buyers and sellers to pick the most attractive auction, let us consider two competing auctions, each of which has a set of buyers and sellers. We also assume that these are the only two available auctions, and a hypothetical buyer must choose between one of the two. Such a binary model can be directly generalized to the case of multiple auctions using the binary model recursively. In Figure 1, we have illustrated the defection model in which we have illustrated two hypothetical auctioneers. Two separate markov models are illustrated. The model on the left (Figure 1(a)) illustrates the defection behavior of a hypothetical buyer. Thus, the hypothetical buyer may shift between states 1 and 2. These states correspond to his choosing auctions 1 or 2 respectively. Thus, an auction buyer may use auction 1 for a current transaction, but may transition to auction 2 for the next transaction with probability p_{12}. Alternatively, they may choose to use the same auction with probability p_{11}. The latter case is illustrated as a self loop in state 1 of Figure 1(a). It is also clear that since the buyer uses either auction 1 or auction 2 for each transaction, we have:

$$p_{11} + p_{12} = 1, \; p_{21} + p_{22} = 1 \tag{4}$$

In general, we would like to find the steady-state probability that the buyer chooses one of the two auctions. We assume that the steady state probability of states 1 and 2 are denoted by α_1 and α_2 respectively. Then, for the Markov model to be in steady state, the transition probability into a given state must be equal to the transition probability out of it. For state 1, we have the following relationship:

$$\alpha_1 \cdot p_{11} + \alpha_1 \cdot p_{12} = \alpha_1 \cdot p_{11} + \alpha_2 \cdot p_{21} \tag{5}$$

This relationship is equivalent to the following:

$$\alpha_1 \cdot p_{12} = \alpha_2 \cdot p_{21} \tag{6}$$

We can set up a similar steady state relationship for state 2, though the relationship is equivalent to that of Equation 6. We also note that the sum of the steady state probabilities over the buyer (or seller) side Markov Model must be equal to 1. Therefore, we have:

$$\alpha_1 + \alpha_2 = 1 \tag{7}$$

We note that we can set up similar relationships at the seller side as well. Let us assume that the steady state probability for states 1 and 2 at the seller side are denoted by β_1 and β_2 respectively. Correspondingly, we have the following relationships among the transition and steady state probabilities:

$$q_{11} + q_{12} = 1,\ q_{21} + q_{22} = 1 \tag{8}$$

$$\beta_1 \cdot q_{12} = \beta_2 \cdot q_{21} \tag{9}$$

So far, we have not connected the state probabilities of the Markov Models corresponding to Figures 1(a) and (b). We earlier observed that the likelihood of the defection of a buyer is dependent upon the number of sellers present at an auction and vice-versa. This effectively means that the transition probabilities of the buyer-model are dependent on the state probabilities of the seller-model and vice-versa. This is a unique way of relating two markov models, since the transition probabilities of one model depend upon the state probabilities of the other and vice-versa. This relationship is defined by the following two non-increasing functions $f(\cdot)$ and $g(\cdot)$:

$$p_{12} = f(\beta_1),\ p_{21} = f(\beta_2),\ q_{12} = g(\alpha_1),\ q_{21} = g(\alpha_2) \tag{10}$$

We refer to these functions as *defection functions*. The function $f(\cdot)$ is the *buyer defection function*, and the function $g(\cdot)$ is the *seller defection function*. The functions $f(\cdot)$ and $g(\cdot)$ should satisfy the following properties:

- $f(\cdot)$ and $g(\cdot)$ are both non-increasing functions. This corresponds to the fact that a higher state probability at the buyer side of an auction corresponds to a lower probability of seller defection from that auction and vice-versa.

- The functions are defined over the probability range $(0, 1)$ and satisfy the following end-point relationships:

$$f(0) = 1,\ f(1) = 0,\ g(0) = 1,\ g(1) = 0$$

These constraints correspond to the fact that a buyer will not participate in any auction which has no sellers and vice-versa.

- The above-mentioned principle can be generalized further to use a *minimum critical mass* in order to define a threshold at which the buyers or sellers will not participate. This minimum critical mass is required for an auction to be a viable activity. We denote this minimum critical mass for the buyers and sellers by c_b and c_s respectively. Below this critical mass, a buyer or seller in the auction has 100% defection probability.

First, we will begin by defining a simple linear functional form for relating the probability of defection to the steady state probabilities. This linear function $f(x)$ is defined in the probability range $(0, 1) \Rightarrow (0, 1)$ as follows:

$$f(x) = \begin{cases} 1 & 0 \leqslant x \leqslant c_s \\ (1 - c_s - x)/(1 - 2 \cdot c_s) & c_s \leqslant x \leqslant 1 - c_s \\ 0 & 1 - c_s \leqslant x \leqslant 1 \end{cases} \tag{11}$$

The corresponding function $g(x)$ is defined similarly except that the critical mass c_b for buyers is utilized. Therefore, we have:

$$g(x) = \begin{cases} 1 & 0 \leqslant x \leqslant c_b \\ (1 - c_b - x)/(1 - 2 \cdot c) & c_b \leqslant x \leqslant 1 - c_b \\ 0 & 1 - c_b \leqslant x \leqslant 1 \end{cases} \tag{12}$$

We note that the above simple function is a natural choice based on the constraints discussed earlier. We will analyze the behavior of the Markov model using this function. We make the following observations:

Observation 2.1 *A steady state solution to the markov model is* $\alpha_1 = 1$*,* $\alpha_2 = 0$*,* $\beta_1 = 1$*, and* $\beta_2 = 0$.

We note that because of the values of α_1 and β_1, the values of the transition probabilities are as follows: $p_{11} = 1$, $p_{12} = 0$, $p_{21} = 1$, $p_{22} = 0$. By substituting these values, we can satisfy all the conditions discussed above. Similarly, we can show the following results:

Observation 2.2 *A steady state solution to the markov model is* $\alpha_1 = 0$*,* $\alpha_2 = 1$*,* $\beta_1 = 0$*, and* $\beta_2 = 1$.

Observation 2.3 *A steady state solution to the markov model is* $\alpha_1 = 0.5$*,* $\alpha_2 = 0.5$*,* $\beta_1 = 0.5$*, and* $\beta_2 = 0.5$.

We note that for general functions $f(\cdot)$ and $g(\cdot)$, the following relationships hold true:

Lemma 2.1 *A steady state solution to the problem can be defined if and only if a pair* $(\alpha_0, \beta_0) \in (0, 1)$ *can be found for which the buyer and seller defection functions satisfy the following relationship:*

$$\alpha_0 \cdot f(\beta_0) = (1 - \alpha_0) \cdot f(1 - \beta_0), \; \beta_0 \cdot g(\alpha_0) = (1 - \beta_0) \cdot g(1 - \alpha_0) \tag{13}$$

Proof: First, let us consider the case when such a pair (α_0, β_0) can be found satisfying the conditions of Equations 13. A steady state solution can be determined by setting $\alpha_1 = \alpha_0$, $\alpha_2 = 1 - \alpha_0$, $\beta_1 = \beta_0$, $\beta_2 = 1 - \beta_0$, $p_{21} = f(1 - \beta_0)$, $p_{12} = f(\beta_0)$, $p_{11} = 1 - p_{12}$, $p_{22} = 1 - p_{21}$, $q_{21} = g(1 - \alpha_0)$, $q_{12} = g(\alpha_0)$, $q_{11} = 1 - q_{12}$, $q_{22} = 1 - q_{21}$. It is easy to verify that all the steady state conditions for the markov model are satisfied by these conditions. For example, the conditions illustrated in Equations 6 and 9 are satisfied because of the pre-conditions in the problem statement of this lemma. The conditions interrelating the two models are satisfied because of the way in which the transition probabilities are chosen.

Next, if we consider a Markov Model in steady state, then the values of (α_0, β_0) can be chosen as the (respective) buyer and seller state probabilities of one of the two auctions. From the steady state conditions, it can be shown that the pre-conditions of the lemma are satisfied. ∎

We note that one of the consequences of this lemma is that it provides conditions on the existence of a steady state in the markov chain. *Furthermore, any steady state can be satisfied by the pair* (α_0, β_0) *which satisfy the above conditions.* These conditions can also be used to show the following:

Corollary 2.1 *For linear defection functions with non-zero critical mass, the only steady state pairs are defined by* $(\alpha_0, \beta_0) = (0, 1)$*,* $(\alpha_0, \beta_0) = (1, 0)$*, and* $(\alpha_0, \beta_0) = (0.5, 0.5)$.

Proof Sketch: It is straightforward to verify our earlier observations that the specified pairs $(0, 1)$, $(1, 0)$, and $(0.5, 0.5)$ are steady state conditions. In order to prove the reverse, let us pick values (α_0, β_0) of the probability which are not equal to $(0, 1)$, $(0.5, 0.5)$, and $(1, 0)$ respectively. We will show that it is not possible to pick values of (α_0, β_0) which are such that Equation set 13 are satisfied. Different cases can be considered for the ranges of α_0 and β_0 with respect to the corresponding critical mass. For each of these cases, it can be shown that the Equation set 13 cannot be consistently satisfied by such a pair.

2.1 Stable and Unstable Equilibrium

We note that not all steady state conditions are equivalent in terms of stability. Intuitively, a steady state condition is stable when a small disturbance to the probability of that state results in the markov model returning to the earlier state of equilibrium. We note that the issue of stability arises in our application because of the special relationship between two Markov Models: the state probabilities of one depend upon the transition probabilities of the other, and vice-versa. In the standard Markov Model, all steady states are stable and vice-versa. In order to define the concept of stability of the steady state in markov models more exactly, we need to define a *dynamic version* of the markov model. In the dynamic version of a markov model, we define *temporally layered states*, in which the ith layer corresponds to the ith transition. Temporally layered markov models are a useful technique for understanding the transient behavior of markov models, and the rate at which a given markov model will reach equilibrium.

For each state i of the standard markov model, the temporally layered markov model has a state i_T for the state i after T transitions. The state i_0 corresponds to the initial probability of state i. For each edge (i, j) in the initial Markov Model, we have a state (i_t, j_{t+1}) in the transformed Markov Model. The value of t can range from 0 to ∞. An example of the dynamic Markov Model is illustrated in Figure 2. The state probability *at time period* t on the buyer side for nodes i_t^1 and i_t^2 are denoted by α_1^t and α_2^t respectively. We note that the state probabilities of the node layer t at all time periods other than at time period t, are equal to zero. This is because of the layered structure of the Markov Model in which a transition occurs into layer t only after t time periods. The corresponding state probabilities on the seller side are denoted by β_1^t and β_2^t respectively. The transition probability of the edge (i_t, j_{t+1}) on the buyer side is denoted by p_{ij}^t. The corresponding transition probability for the seller side of the Markov Model is denoted by q_{ij}^t. As in the static Markov Model, the transition probabilities are related to the state probabilities. In this case, the transition probabilities at time period t at the buyer side are related to the state probabilities at time period t and vice-versa.

$$p_{12}^t = f(\beta_1^t),\ p_{21}^t = f(\beta_2^t),\ q_{12}^t = g(\alpha_1^t),\ q_{21}^t = g(\alpha_2^t) \tag{14}$$

The corresponding transition equations in the Markov Model are defined as follows:

$$\alpha_1^t \cdot p_{11}^t + \alpha_2^t \cdot p_{21}^t = \alpha_1^{t+1} \cdot (p_{11}^{t+1} + p_{12}^{t+1}) \tag{15}$$

Since the sum of the transition probabilities out of a state is 1:

$$\alpha_1^t \cdot p_{11}^t + \alpha_2^t \cdot p_{21}^t = \alpha_1^{t+1} \tag{16}$$

The corresponding transition condition on the seller side is defined as follows:

$$\beta_1^t \cdot p_{11}^t + \beta_2^t \cdot p_{21}^t = \beta_1^{t+1} \tag{17}$$

In general, as $t \to \infty$ the state probabilities α_1^t and α_2^t will move to one of the steady states. In the standard form of the Markov Model, only one steady state exists which is independent of the initial state probabilities α_1^0 and α_2^0. However, in the form of the model discussed in this paper, (in which the transition probabilities of one model depend upon the state probabilities of the other), the final state probabilities depend both upon the initial state probabilities as well as the initial state of the Markov Model. In addition, the Markov Model is more likely to rest in a final steady state which is *stable*. Consequently, we will define the concept of stability of steady state. In essence the steady state condition states that a small perturbation from the state distribution is likely to bring it back to its original state.

Definition 2.1 *A steady state* (α, β) *is said to be stable, if for any small perturbation vector* ϵ *such that* $|\epsilon| \leq \epsilon_0$*, a starting state of* $(\alpha_0, \beta_0) = (\alpha, \beta) + \overline{\epsilon}$ *leads to* (α, β) *as the final state. Therefore, we would have:*

$$lim_{t \Rightarrow \infty}(\alpha_t, \beta_t) = (\alpha, \beta). \tag{18}$$

The intuitive significance of the above definition is that a small perturbation from the steady state is likely to lead to the system reverting back to its steady state. The aim of this is to model real life situations in which minor transitory events lead to the system being perturbed from its steady state. In such cases, stable steady states are more likely to reflect the final behavior of the model. We make the following conjectures about the behavior of these models:

Conjecture 2.1 *The solution* $(\alpha, \beta) = (0.5, 0.5)$ *is not stable.*

The intuition behind this conjecture is that a reduction of the state probabilities below 0.5 for a particular auction on the buyer side reduces the state probability at the seller side as well. (This is because of the functional relationship between the state and transition probabilities.) the corresponding transition probabilities on the seller side and vice-versa. This leads to a self sustaining cycle of reduction on state probabilities for one of the two auctions. We currently do not have a formal proof of this behavior, but will illustrate the process through simulation. Other observations about the state probabilities are as follows:

Observation 2.4 *The solutions* $(\alpha, \beta) = (0, 1)$ *and* $(\alpha, \beta) = (1, 0)$ *are stable.*

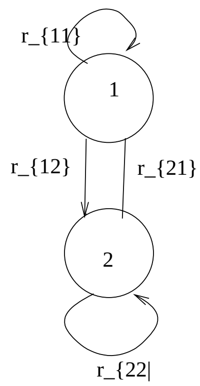

Figure 3: Markov Model for the Social Network

This observation is straightforward, because any perturbation which is lower than the critical mass reverts the system to the steady state in the very next iteration.

The implications of the above observations are that the auction system is in a *stable* steady state in case of one or the other auction dominating. In the next section, we will provide some empirical simulations which illustrate the extent of the domination of the states.

2.2 Adding Temporal Memory to the Model

We note that the transition probabilities on the buyer side during a given time period are dependent upon the state probabilities on the buyer side and vice-versa. In the model discussed above, we have assumed that the transition probabilities during time period t are dependent upon the state probabilities during the same time period t. This is purely a *memoryless* model, since the state probabilities during earlier periods do not affect the transition probability. In practice, there may be a delay in the process of buyers or sellers reacting to state changes in the Markov Model. Therefore, we define a *time averaged* state probability as follows:

$$\alpha_1'^t = \frac{\sum_{i=i}^{t} \alpha_1^i \cdot 2^{i-t\cdot\lambda}}{2^{i-t\cdot\lambda}} \tag{19}$$

The value of $1/\lambda$ is defined as the half-life during which the importance of a state probability gets reduced by half. We note that for very large values of λ, this corresponds to the purely memory less model. The transition probabilities during the time period t are defined as a function of the time-averaged state probabilities during the same time period.

$$p_{12}^{t} = f(\beta_1'^{t}),\ p_{21}^{t} = f(\beta_2'^{t}),\ q_{12}^{t} = g(\alpha_1'^{t}),\ q_{21}^{t} = g(\alpha_2'^{t}) \tag{20}$$

We note that the addition of memory to the model generally affects the rate of convergence to the model, but it does not usually affect the state to which the system converges. A lower value of λ increases the half life for calculating the relationship between the transition probabilities and the state probability of the different Markov Models. This also increases the time required by the model to reach steady state. In the next section, we will also study the effect of choosing different values of the parameter λ.

3 Analysis for Social Networks

In this section, we will discuss the network effect for applications in which any pair of participants may interact with one another. This is generally the case for socially focussed applications such as chat rooms and social networks. For ease in discussion, we will assume the case of the social networking application, though these results can also be generalized to arbitrary applications. In the case of a social network, the interaction may occur between any pair of nodes, and the probability of a person joining a social network is highly dependent upon the probability of his friends being a part of the social network as well. This is in turn dependent upon the probability that different individuals become a part of the social network as well.

In order to build the Markov Model, we will make a number of assumptions. Unlike the auction model, one of the key differences is that it is a possible for an individual to belong to multiple social networks. However, at a given time, a person is active only in one network. Therefore, we assume the following:

- At a given time, a person is active only in one social network.

- The probability of a person being active on the social network at a given time is dependent upon the probability that his or her friends are active on it.

We note that even though individuals may register for multiple social networks, they may stay active on only one for long periods of time. The aim of this analysis is to show that there is a high likelihood that a majority of the participants may be active on only one of the social networks. The result of this is that one of the social networks becomes progressively more active, and the remaining become less active.

In order to formally model the propensity of users to pick the most relevant social network, let us consider two competing networks, and a user may pick any one of them. Correspondingly, we create a Markov chain with two states. The two states correspond to a hypothetical user being active in social network 1 and social network 2. Note that we have only one markov model in this case, since we do not distinguish between buyers and sellers in this model. In Figure 3, we have illustrated the defection model in which we have illustrated two hypothetical users of the social network. A hypothetical user may shift between states 1 and 2. These states correspond to his being active in social networks 1 and 2 respectively. The probability of an individual moving between the two states are denoted by r_{12} and r_{21} respectively. The probability of an individual staying in the same state are denoted by r_{11} and r_{22} respectively.

Let γ_1 and γ_2 be the steady state probabilities of the two states. Then, the steady-state relationships between the two states are as follows:

$$\gamma_1 \cdot r_{12} = \gamma_2 \cdot r_{21}$$

$$\gamma_1 + \gamma_2 = 1$$

We note that γ_1 and γ_2 also represent the steady state probabilities for an individual being in networks 1 and 2 respectively. The transition probabilities are correspondingly defined as follows:

$$r_{11} + r_{12} = 1$$

$$r_{21} + r_{22} = 1$$

Next, we examine how the state and transition probabilities of the Markov Model are connected to one another. We note that the defection of a user from one social network to the other is dependent upon the corresponding steady state probability. This is because a social network with a larger number of users is likely to have a lower defection rate and vice-versa. This relationship is defined by the non-increasing functions $h(\cdot)$:

$$r_{12} = h(\gamma_1),\ r_{21} = h(\gamma_2) \tag{21}$$

The function $h(\cdot)$ is referred to as a defection function. This function is a non-increasing function because a higher state probability at a social network corresponds to a lower probability of transition out of it. As in the case of the auction model, the function $h(\cdot)$ is defined over the probability range $(0, 1)$ and satisfies the following end-point relationship:

$$h(0) = 1,\ h(1) = 0$$

This corresponds to the fact that a user will not stay in an empty social network. Similarly, a user will not leave a social network for one which does not have any other user.

The above-mentioned principle can be generalized further to use a *minimum critical mass* in order to define a threshold at which user will join a social network. If the social network has fewer than a certain fraction of the users, it will not stay viable, and all users will defect from it. We denote this minimum critical mass by c_n. Below this critical mass, a user will always move to the other social network.

As in the previous case, we will define a linear functional form for relating the probability of defection to the steady state probabilities. This linear function $h(x)$ is defined in the probability range $(0, 1) \Rightarrow (0, 1)$ as follows:

$$h(x) = \begin{cases} 1 & 0 \leqslant x \leqslant c_n \\ (1 - c_n - x)/(1 - 2 \cdot c_n) & c_n \leqslant x \leqslant 1 - c_n \\ 0 & 1 - c_n \leqslant x \leqslant 1 \end{cases} \tag{22}$$

We make the following observations as in the case of the auction model. The arguments are similar to those of the case of the auction model. In fact, the arguments are simpler in this case, since we are dealing with a single markov model rather than two connected models.

Observation 3.1 *There are three possible steady states to this model:*

- *The first steady-state probability is* $\gamma_1 = 0$*,* $\gamma_2 = 1$*.*
- *The second steady-state probability is* $\gamma_1 = 1$*,* $\gamma_2 = 0$*.*
- *The third steady state probability is* $\gamma_1 = 0.5$*,* $\gamma_2 = 0.5$

Of the three steady states, only two of them are stable. Therefore, we have;

Observation 3.2 *Only two of the three steady states are stable. Specifically, the third steady state* $\gamma_1 = 0.5$*,* $\gamma_2 = 0.5$ *is unstable.*

These results are completely analogous to those for the case of the auction model. Next, we will provide experimental results which show the effect on both models. Furthermore, as in the case of the auction model, we can ass temporal memory to the social networking model with the use of the parameter λ. Since the extension is very straightforward and can be done in an exactly analogous way to the auction model, we omit a more detailed description.

4 Implications for Network Management

The results have a number of implications for the actual management of the network-based applications. The key implication is primarily for the case of networks which need to grow in the presence of already existing dominant players. While the results of this paper seem to suggest that the presence of a dominant player should be a deterrent to the entry of other players, it turns out that the results of this paper also have key implications for understanding the conditions that enable the effective entry of secondary players in the field.

The key assumption for designing the competing models is that the interaction behavior of different players *is sufficiently similar so as to enable effective comparison of the different networks by the participants*. This is because the participants can make reasonable decisions about defection behavior between different networks (based on the number of interacting participants), only if the mode of interaction between the participants is very similar. This suggests that the best route for entry of new players in any given network application domain in a presence of an existing dominant player is the use of a completely different interaction model. We present some examples of how a network can distinguish itself with the use of different kinds of features, which will appeal to participants from different domains.

- An auction site which is designed with a sufficiently different bidding model than a currently existing site is likely to attract a slightly different segment of the participants, because of the different experience associated with the auction itself. For example, penny auctions *charge participants for the option to bid* by raising the current bid by a penny. The auction makes money which is generated by the potentially large number of bids entered by different participants, though the final purchase price for the winner is a small fraction of the true price of the item. In effect, the other bidders subsidize the price of the item for the final winner. This provides a fast-paced "game aspect" to the auction which is enjoyable to the participants and very attractive to certain kinds of players in spite of the presence of a dominant conventional site such as *Ebay*.[8]

- Different social networking sites may have different kinds of connections which may lead to different kinds of interactions. This may make them less prone to direct competition with one another for participants, and may allow co-existence of participants across different

[8]For example the site *QuiBids* (www.quibids.com) has been very successful. The final prices of items are often less than 5% of their true market prices.

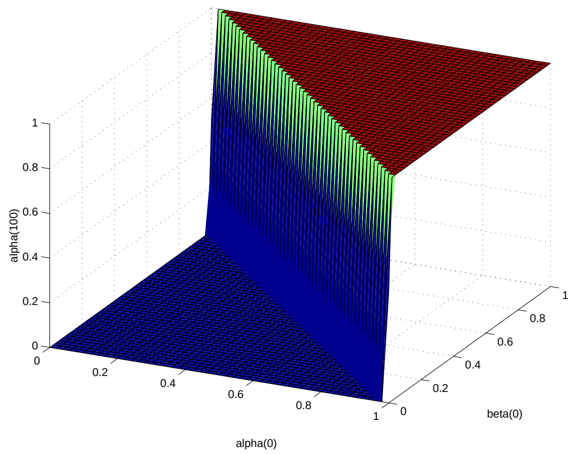

Figure 4: Final State of Model with Different Starting States

sites. For example, the concepts of followers and follower-centered tweets is quite different from the modes of connections and communications on a site like *Facebook*. Similarly, a professional site such as *LinkedIn* may allow for employer references and other professional services which are not available at many other sites. This difference in site management and features allows for the existence of multiple sites simultaneously.

In addition, the incorporation of *interoperability* with existing networks is an effective methodology in order to negate the existing network effect of competing sites. For example, the Google *Social Circle* application builds upon the *Facebook* connections of an actor in order to provide leverage in the social network construction process. Thus, the addition of interoperability between competing sites is one way for a secondary provider to quickly build upon the existing network of the dominant network. In general, the goal is to design the network, so as to bypass the assumption of direct competition between the different sites either on the basis of differentiation or interoperability.

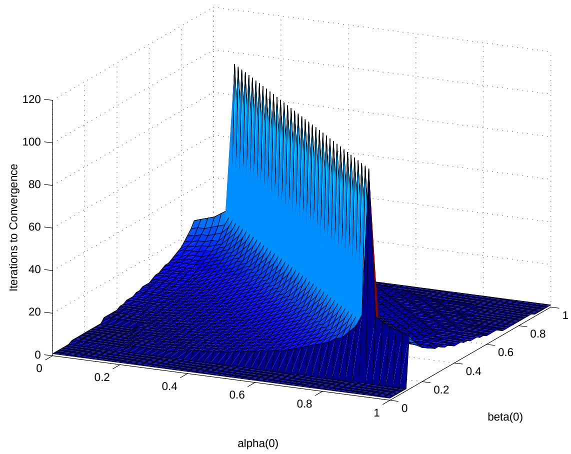

Figure 5: Number of Iterations to Convergence with Different Starting states

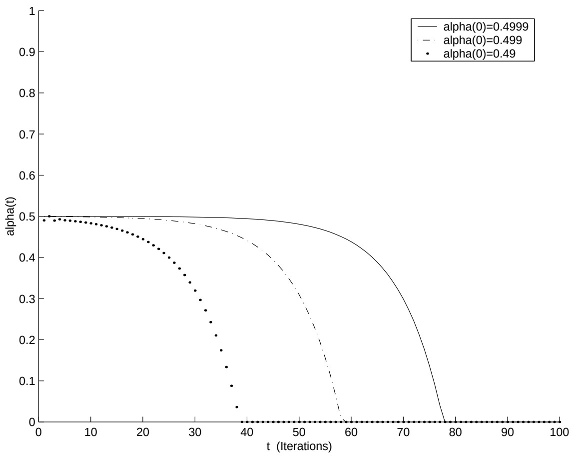

Figure 6: Progression of State Probabilities with Different Starting States

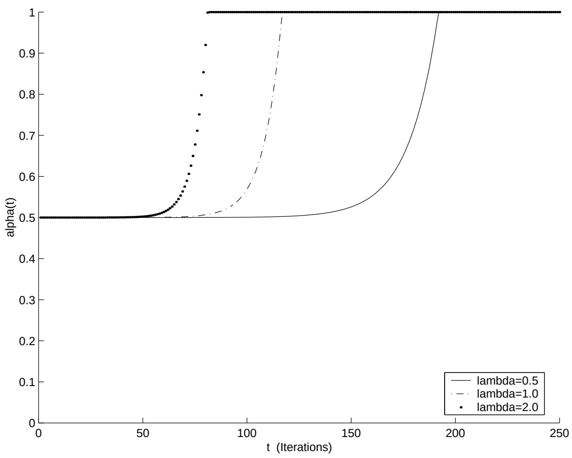

Figure 7: Progression of State Probabilities with Different Memory Parameters

5 Experimental Simulations

In this section, we will illustrate some experimental simulations which confirm the convergence behavior of the markov model for online auctions and social networks. We will also illustrate how different starting points affect the behavior of the convergence towards a steady state. First, we will simulate the case of online auctions. Then, we will study the case of social networks.

5.1 Simulation for Online Auctions

We construct a simulation of the Markov Model using MATLAB software. The use of the dynamic variation of the model allowed us to test the behavior of the rate of convergence as well as the effect of the initial states. Since real data about defection behavior in social networks and auction networks is hard to obtain, we used a simulation for our study. An important issue to be examined is how the initial choice of state probabilities affects the final state probabilities in the model. In Figure 4, we have illustrated the behavior of the final state probabilities (after 100 transitions of the Markov chain) with different initial state probabilities. In this case, the parameters were set at $c_b = c_s = 0.1$ and $\lambda = 1$. The value of (α, β) was allowed to vary over the entire range of possibilities in the unit square over $50 * 50 = 2500$ points in the grid. It is interesting to notice that over almost the entire range of values, the model converged to either (0,0) or (1, 1). For example, even when using the value of $\alpha_0 = 0.5$, $\beta_0 = 0.48$, the system converged to the value $(0, 0)$. Intuitively, this means that even if one of the two auctions had a slight advantage over *either* the buyer or seller side, this advantage is sufficient for one auction to overwhelm the other. This is evidence of the fact that the state $(0.5, 0.5)$ does not correspond to a stable steady state. The only set of initial states (α_0, β_0) for which the system did not always converge to either $(0, 0)$ or $(1, 1)$ was the set of values in the grid along the line $\alpha_0 + \beta_0 = 1$. As is evident from Figure 4, for the entire range of values along each side of this dividing line, the final state of the model takes on the value $(0, 0)$ or $(1, 1)$. The only set of values which converged to $(0.5, 0.5)$ were found

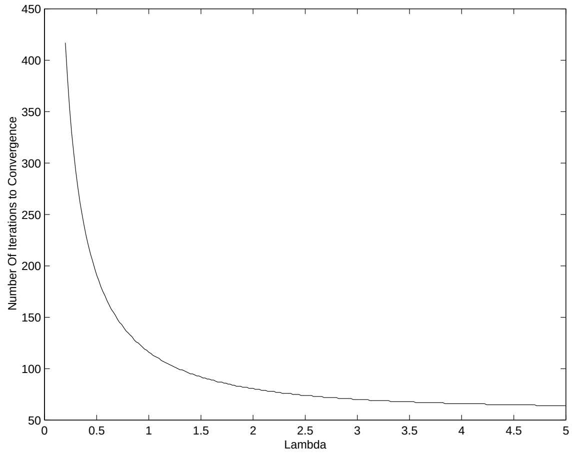

Figure 8: Rate of Convergence with different memory parameters

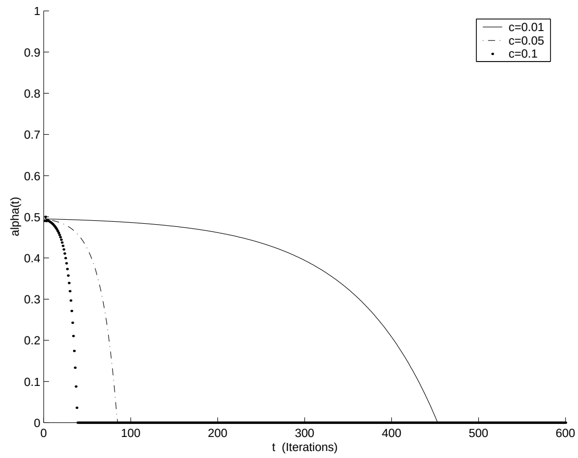

Figure 9: Progression of State Probabilities with Different Critical Mass Thresholds

along the dividing line. We also note that in some cases, small rounding errors lead to the system converging to $(0, 0)$ or $(1, 1)$, whereas the final state ought have been $(0.5, 0.5)$. This is again an evidence of just how unstable the state $(0.5, 0.5)$ is in practice, since such errors are also likely to be manifested as probabilistic variations in real life. Thus, this suggests that even with a relatively even distribution between networks at the beginning, one of the networks is very likely to dominate the other one at the end. In many cases, 100 transitions were not sufficient to reach convergence at this point. This brings to the natural observation that a greater level of skew in the initial starting states required a greater level of iterations for convergence. Next, we will study the effect of the initial starting state on the number of iterations required for convergence.

In Figure 5, we have illustrated the variation in the number of iterations with different initial starting states. In this cases, we defined convergence as the minimum number iterations required for one of the states to reach within 0.001 of the final steady state. As in the previous case, the positions in the grid of initial values for (α_0, β_0) which are closer to the line $\alpha_0 + \beta_0 = 1$ took a longer time to converge than the points which were closer to $(0, 0)$ or $(1, 1)$. Intuitively this means that while a small perturbation leads to one or the other auctions dominating, the rate of convergence is heavily dependent upon the initial state. We also tested a few data points which lay within 10^{-6} of the steady state $(0.5, 0.5)$. In such cases, the system required more than 100 iterations to converge to the final value. In order to illustrate the effect of the initial starting state a little better, we have illustrated the behavior of the convergence with the number of iterations for different starting points in Figure 6. In each case, the value of β was chosen to be 0.5, whereas the value of $alpha$ was set to the three different values of $\alpha = 0.49, 0.499, 0.4999$. It is clear that the rate of convergence was heavily affected by the initial state, though the final state was the same in all three cases. The closer the initial state was to 0.5, the longer it took to converge to the final state. In the context of a real application, it means that a relatively even size of the network at the initial phase results in a slower rate of dominance of one network over the other.

We tested the convergence behavior for different values of the memory parameter λ. In Figure 7, we have illustrated the progression of the state probabilities for different values of λ. In each case, we chose an initial state probability of $\alpha_0 = 0.50001$, $\beta = 0.5$, and $c_b = c_s = 0.1$. It is clear that a lower value of λ leads to slower convergence, but it does not affect the final state to which the system converges. This situation is illustrated more explicitly in Figure 8 in which we have plotted the total number of iterations to convergence for different values of the memory parameter λ. The definition of convergence is the same as discussed earlier. These results illustrate the natural conclusion that a higher amount of persistent memory leads to a slower and more stable convergence, but does not affect the final disposition of the stable steady state. We note that a higher level of memory corresponds to more persistent customers who take a longer time to realize the benefits of using one auction over another. Correspondingly, the time required for one of the

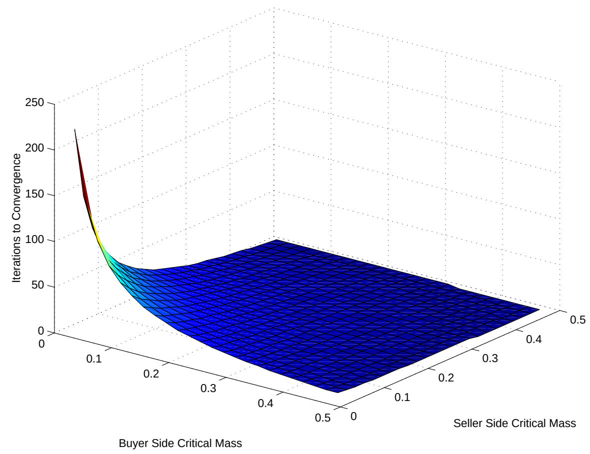

Figure 10: Number Of Iterations to Convergence with Different Critical Mass Thresholds

auctions to dominate the other is also greater. Finally, we tested the effect of the critical mass on the convergence behavior. Our primary observation from these tests was that the critical mass did not affect the final state to which the system converged. However, it did affect the rate of convergence. In Figure 9, we have illustrated an example in which we have tested the progression of the steady state probability for different values of the critical mass. In this case, the value of the critical mass on both the buyer and seller side were set to the same value denoted by c. The values of α_0 and β_0 were set at 0.49 and 0.5 respectively. It is clear that in each case, the value of α_t finally converged to 0 (corresponding to auction 2 dominating as it had the initial advantage). In order to determine the convergence behavior more explicitly, we calculated the number of iterations required for convergence for different combinations of the buyer and seller side critical masses c_b and c_s. To do so, we created a $24 * 24$ grid corresponding to the ranges [0.02, 0.48] for the buyer and seller side critical masses. In each case, we used a starting state of $\alpha_0 = 0.51$, $\beta_0 = 0.5$. The results are illustrated in Figure 10. It is clear that the lower the critical mass, the slower the convergence of one of the auctions. This is a natural consequence of the fact that a lower critical mass also makes it more difficult for one of the auctions to dominate the other. However, it did not affect the final result in terms of the identity of the auction that dominated the other. From the perspective of interaction-based networks such as auctions, the critical mass plays a key role in the sustainability of the social network.

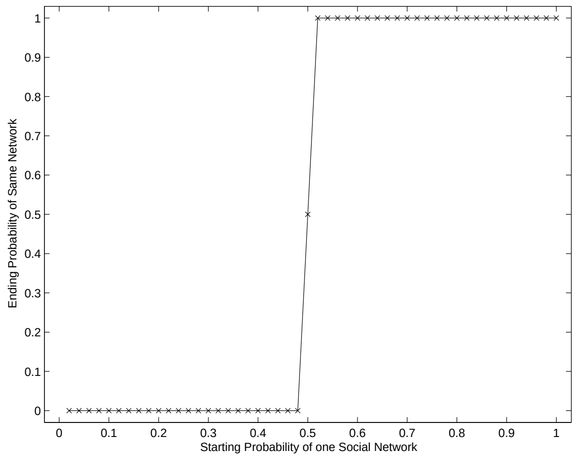

Figure 11: Final State of Social Network Model with Different Starting States

5.2 Results for Social Network Model

In Figure 11, we have illustrated the behavior of the final state probabilities of the social network model after 100 transitions of the Markov chain. We used different initial state probabilities for γ. The value of γ was allowed to vary fully in the range $[0, 1]$. The other parameters were set at $c_n = 0.1$ and $\lambda = 1$. It is interesting to notice that over almost the entire range of values, the model converged to either 0 or 1. It was only for the case of a starting probability of 0.5 that the system converged to 0.5 as well. We note that even a slight variation from a starting state of 0.5 lead to a convergence to one of the other two states. This shows that this state is unstable. In real networks, a precisely pair of balanced networks is unlikely, and therefore one or the other network is more likely to dominate. Next, we will examine the number of iterations it takes to reach convergence for different starting states.

In Figure 12, we have illustrated the variation in the number of iterations with different initial starting states. As in the previous case, we defined convergence as the minimum number iterations required for one of the states to reach within 0.001 of the final steady state. We further note that for a starting state of 0.5, the method did not converge after 500 iterations, and we terminated at that point. Therefore, this particular coordinate was set at 500, but it does not imply convergence in the figure. Clearly, it is evident that the closer the social network is to an initial starting state

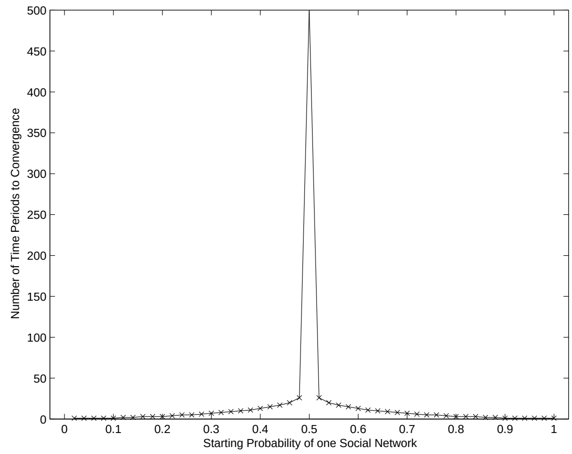

Figure 12: Number of Iterations to Convergence of Social Network Model with Different Starting States

of 0.5, the slower the convergence. This is a natural result, since it suggests that a greater skew in the initial probability distribution across the two social networks will lead to faster convergence. Thus, a small perturbation leads to one or the other social networks dominating, and the rate of convergence is heavily dependent upon the initial state. Therefore, a perturbation in a real social network which draws members of one social network into another is likely to speed up the process of one of the networks dominating the other.

6 Conclusions and Summary

In this paper, we provided an analysis of the network effect in two key kinds of web applications. In one kind, interactions occur between distinct classes of entities, whereas in another, interactions occur between the same class of entities. As examples, we model interactions in auctions and social networks. We note that many other applications can be modeled in a similar way:

- The use of chat-rooms or forums has a very similar model as the social networking model. This is also true of electronic commerce applications such as collaborative filtering (Shardanand and Maes, 1995).

- The use of any form of collaborative networking application may be modeled in a similar way as one of the two cases depending upon whether or not the users are heterogeneous.

The final results are likely to vary across different applications because of the different importance of the network effect in different systems. This may show up in the form of varying rates of convergence or partial co-existence of different applications. However, the effect of this is likely to be extremely powerful over time in most cases. For example, in the case of auctions, we already see overwhelming evidence of a single dominant player.

The primary limitation of the model is its use of simplifying assumptions in terms of the user being able to make efficient decisions decisions between two virtually identical networks, where all other factors remain the same. In practice, the markets may not be completely efficient or the competing networks may have sufficient differences in terms of features and usage which can make a difference aside from the factor of network size. Nevertheless the model does provide a good understanding of the dynamics of defection with respect to network size.

One of the goals of this work is to provide an understanding of the cases where the insights gleaned from the work can be used by a network administrator. For example, the assumptions which are necessary to guarantee the network effect, also provide an understanding of cases, where the network effect does not hold. This provides an understanding to the administrator of a new network about how it may be grown in the presence of another dominant network.

In future work, we will study the network effect in the context of models which are not completely efficient, and the networks are asymmetric in terms of their level of attraction to different users. We will study the interplay of these different factors and the dynamics of multiple networks in such scenarios.

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